# What MDL can bring to Pattern Mining

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## Introduction

**Patterns** are subsets of attributes that describe an object.

**Pattern Mining. Objective:** find a small set of patterns that are well interpretable by experts.

**Input data:** binary table G x M, where G is a set of objects, M is a set of attributes, and I is a relation between them.

Interpretation of gIm: object $g \in G$ has attribute $m \in M$.

## Background Knowledge: Assumptions on Interestingness

Idea: use measures that reflect knowledge of experts about ‘interestingness” of patterns

Examples of interestingness measures for concept *(A,B)*

[area] “interesting patterns are those that take the biggest area in dataset”

[length] “Interesting patterns are the most detailed ones that are quite frequent in dataset where $I(\cdot)$ is the indicator*, *q* is a threshold.

[separation] “Interesting patterns are separated the best from the context”

combined measures, etc.

$$area(A,B) = |A| \cdot |B|$$

$$length(A,B) = |B| I(|A| \geq q)$$

$$sep(A,B) = \frac{|A||B|}{\sum_{g \in A}|g'| + \sum_{m \in B}|m'| - |A| \cdot |B|}$$

$$^{*} \quad I(cond) = \begin{cases} 1 & \textbf{if } cond \textbf{ is } True \\ 0 & \textbf{otherwise} \end{cases}$$

## Pattern Mining. What kind of patterns we should compute?

Total number of patterns is $2^M$

### Types of patterns in terms of Formal Concept Analysis

### FCA. Basic Notions

A **formal context** [Ganter and Wille, 1999; Wille, 1982] is a triple *(G,M,I)*, where G is a set objects, *M* is a set attributes, $I \subseteq G \times M$ is a relation called **incidence relation.**
The **derivation operator** $(\cdot)'$ is defined for $Y \subseteq G$ and $Z \subseteq M$ as follows:

$$Y' = \{ m \in M \mid gIm \text{ for all } g \in Y \}; \quad Z' = \{ g \in G \mid gIm \text{ for all } m \in Z \}$$

A **(formal) concept** is a pair *(Y, Z)*, where $Y \subseteq G$, $Z \subseteq M$ and $Y' = Z$, $Z' = Y$. *Y* is called the (formal) **extent** and *Z* is called the (formal) **intent** of the concept *(Y, Z)*.

A **concept lattice** (or Galois lattice) is a partially ordered set of concepts, the order $\ll$ is defined as follows: $(Y, Z) \ll (C, D)$ iff $Y \subseteq C$ ($D \subseteq Z$), a pair *(Y, Z)* is a subconcept of *(C, D)* and *(C, D)* is a superconcept of *(Y, Z)*.
Formal concepts ordered by **generality relation** $(A_1, B_1) \ll (A_2, B_2)$ iff $A_1 \subseteq A_2$ make a lattice, called **concept lattice.**

### Types of patterns (defined for concept (A,B)):

**Closed itemises** (intents): *B*.
**Minimal generators** are minimal subsets $B_i \subseteq B : B_i' = A$.
**Generators** are any patterns between minimal generators and closed itemises

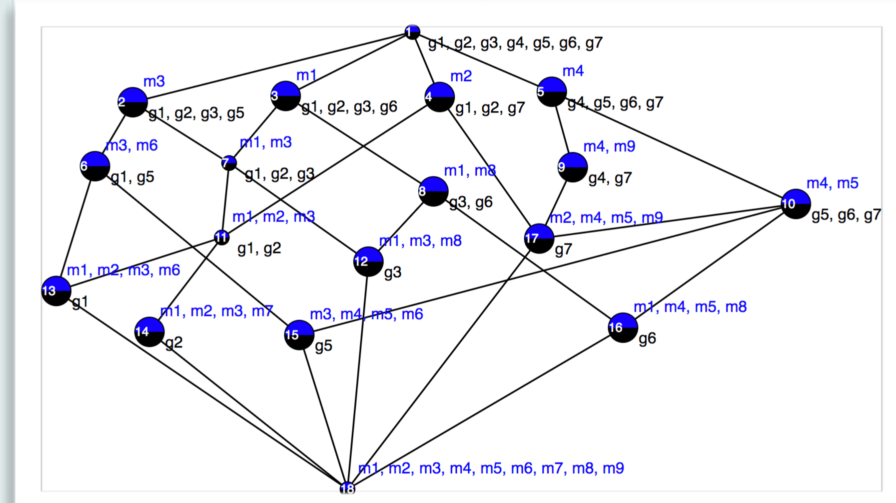
## Example

Formal context

| | Objects | m1 | m2 | m3 | m4 | m5 | m6 | m7 | m8 | m9 |
|---|---|---|---|---|---|---|---|---|---|---|
| g1 | dog | X | X | X | | | X | | | |
| g2 | cat | X | X | X | | | | X | | |
| g3 | frog | X | | X | | | | | X | |
| g4 | car | | | | X | | | | | X |
| g5 | ball | | | X | X | X | X | | | |
| g6 | chair | X | | | X | X | | | X | |
| g7 | fur coat | | X | | X | X | | | | X |

$m_1$: 4 legs
$m_2$: wool
$m_3$: change size
$m_4$: cold-resistant
$m_5$: do release $CO_2$
$m_6$: black-white
$m_7$: yellow-braw
$m_8$: green
$m_9$: gray

Concept lattice (partially ordered full set of formal concepts)



For a formal concept $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$

- closed patterns $\{m_1, m_2, m_3\}$;
- minimal generators $\{m_1, m_2\}$, $\{m_2, m_3\}$;
- generators $\{m_1, m_2\}$, $\{m_2, m_3\}$, $\{m_1, m_2, m_3\}$.

The most interesting concepts w.r.t. given assumptions:

(area) $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$, $(\{g_1, g_2, g_3\}, \{m_1, m_3\})$, $(\{g_5, g_6, g_7\}, \{m_4, m_5\})$; area = 6
(length, frequency ⩾ 2): $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$; length = 3
(separation): $(\{g_1, g_2\}, \{m_1, m_2, m_3\})$, $(\{g_1, g_2, g_3\}, \{m_1, m_3\})$; separation = 6/13.

**Background Knowledge**

**Input data** → **Compute patterns** → **Reorder patterns** → **Filter patterns**

## Minimal Description Length (MDL) Principle. Basic Definitions

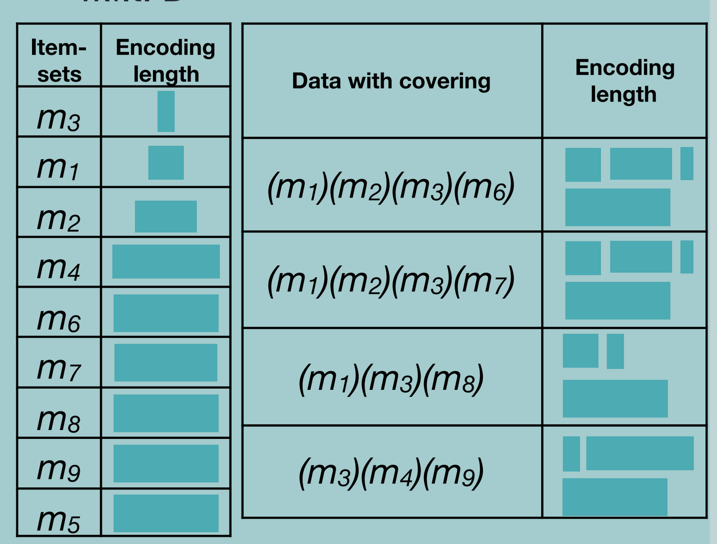
**The main principle:** the best set of patterns is the set that best compresses the database [Vreeken et al., 2011].

**Objective:** $L(D, CT) = L(D \mid CT) + L(CT)$, where $L(D \mid CT)$ is the length of the dataset encoded with the code table *CT* and $L(CT \mid D)$ is the length of the code table *CT* computed w.r.t. *D*.

**Key notions:**

- **Encoding length:** new length that "compresses", i.e. the most frequently used ones have the shortest encoding length.
- **Code table:** a set of selected patterns with their encoding lengths.
- **Disjoint covering:** principle of compression by patterns.

Total length: $L(D, CT) = L(CT \mid D) + L(D \mid CT)$
Code table length w.r.t. data: $L(CT \mid D) = \sum_{i \in CT} code(i) + len(i)$
Data length w.r.t. code table: $L(D \mid CT) = \sum_{d \in D} \sum_{i \in cover(d)} len(i)$




## MDL in practice: greedy algorithm (Krimp)

Initial state

| CT: Itemsets | CT: Usage |
|---|---|
| $m_3$ | 4 |
| $m_1$ | 3 |
| $m_2$ | 2 |
| $m_4$ | 1 |
| $m_6$-$m_7$ | 1 |
| $m_5$ | 0 |

| Data with covering |
|---|
| $(m_1)(m_2)(m_3)(m_6)$ |
| $(m_1)(m_2)(m_3)(m_7)$ |
| $(m_1)(m_3)(m_8)$ |
| $(m_3)(m_4)(m_9)$ |

| Candidate set, area |
|---|
| $m_1m_2m_3$, 6 |
| $m_1m_3$, 6 |
| $m_1m_2m_3m_6$, 4 |
| $m_1m_2m_3m_7$, 4 |
| $m_1m_3m_8$, 3 |
| $m_3m_4m_9$, 3 |

An intermediate state

| CT: Itemsets | CT: Usage |
|---|---|
| $m_1m_2m_3$ | 2 |
| $m_3$ | 2 |
| $m_1, m_4$ | 1 |
| $m_6$-$m_9$ | 1 |
| $m_2, m_5$ | 0 |

| Data with covering |
|---|
| $(m_1m_2m_3)(m_6)$ |
| $(m_1m_2m_3)(m_7)$ |
| $(m_1)(m_3)(m_8)$ |
| $(m_3)(m_4)(m_9)$ |

| Candidate set, area |
|---|
| $m_1m_3m_8$, 3 |
| $m_3m_4m_9$, 3 |
| $m_1m_3$, 2 |

Add ordered candidates one by one if they allow for reducing the total length

Final state

| CT: Itemsets | CT: Usage |
|---|---|
| $m_1m_2m_3$ | 2 |
| $m_1m_3m_8$ | 2 |
| $m_3m_4m_9$ | 1 |
| $m_6, m_7$ | 1 |
| $m_1$- $m_5$ | 0 |
| $m_8$- $m_9$ | 0 |

| Data with covering |
|---|
| $(m_1m_2m_3)(m_6)$ |
| $(m_1m_2m_3)(m_7)$ |
| $(m_1m_3m_8)$ |
| $(m_3m_4m_9)$ |

Reduction in the number of patterns*

| dataset | nmb. of obj. | nmb. of attr. | nmb. of concepts total | nmb. of concepts MDL | dataset | nmb. of obj. | nmb. of attr. | nmb. of concepts total | nmb. of concepts MDL |
|---|---|---|---|---|---|---|---|---|---|
| auto | 205 | 135 | 67 557 | 19.26 | horse colic | 368 | 83 | 173 808 | 101 |
| breast | 699 | 16 | 642 | 9.04 | iris | 150 | 19 | 107 | 13 |
| car | 1 728 | 25 | 12 617 | 94 | led7 | 3 200 | 24 | 1 937 | 152 |
| chess | 28 056 | 58 | 152 753 | 1 675 | mushroom | 8 124 | 90 | 181 945 | 211 |
| dermatology | 366 | 49 | 16 324 | 47 | nursery | 12 960 | 30 | 176 536 | 392 |
| ecoli | 336 | 29 | 694 | 25 | page blocks | 5 473 | 44 | 715 | 45 |
| flare | 1 389 | 38 | 16 303 | 106 | pima indians | 768 | 38 | 1 609 | 50 |
| glass | 214 | 46 | 4 704 | 50 | ticTacToe | 958 | 29 | 42 685 | 160 |
| heart | 303 | 50 | 36 708 | 54 | wine | 178 | 68 | 13 170 | 52 |
| hepatitis | 155 | 52 | 199 954 | 44 | zoo | 101 | 42 | 4 563 | 17 |

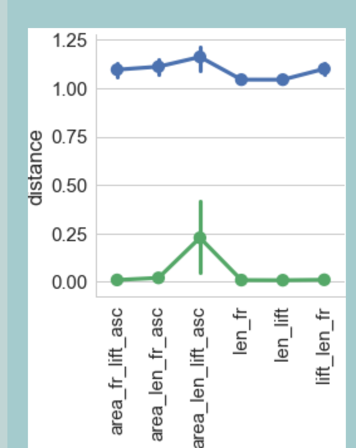
Significant reduction in the number of patterns (up to 5% of the formal concepts).

* datasets from LUCS-KDD repository [4]

## MDL: is there a place for background knowledge?

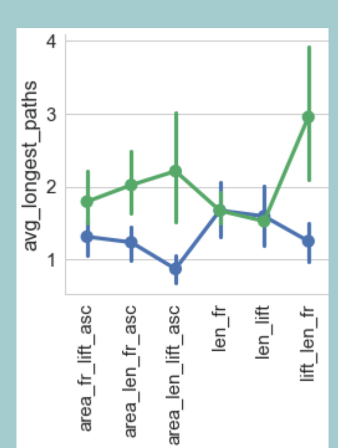
Idea: MDL as an additional filtering stage in pattern selection.

MDL-optimal (blue) vs top-n (green) closed itemsets




**Non-redundancy**
Distance to the 1st NN

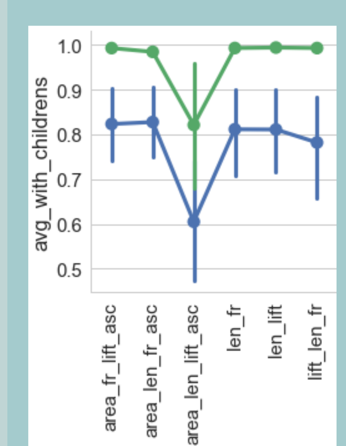
Top-n concepts have a lot of “twins”, while MDL-optimal ones are pairwise distinctive (w.r.t. Euclidean distance).




**Non-redundancy**
Average length of the longest paths built from posets (lattices)

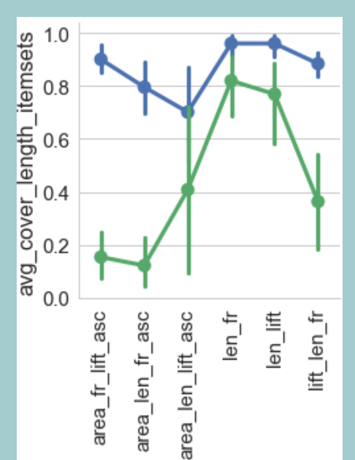
A long path is an indicator of redundancy, since in that case patterns characterize the same objects at different levels of abstraction. Short paths correspond to “flat” structures with more varied patterns.

Pattern mining with area len_sep and area_sep lift, lift_len_fr can be significantly improved by the application of MDL.




**Non-redundancy**
Average number of itemsets with children

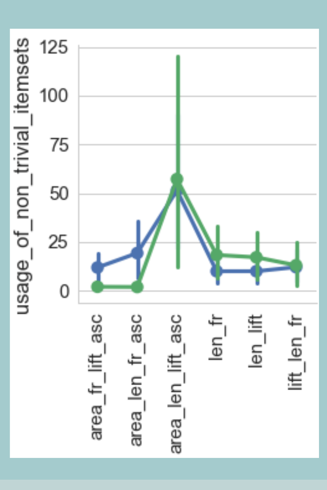
Characterizes the uniqueness of patterns in a set. It indicates just an amount of itemsets having at least one more general itemset.




**Data coverage**
The rate of covered “crosses” in object-attribute relation

A subset of selected patterns can be considered as a concise representation of a dataset. Thus, it is important to know how much information is lost by compression. It can be measured by the rate of covered attributes. Values close to 1 correspond to the lossless compression

MDL ensures better covering and allows for the biggest gain for area-based orderings.




**Typicality (representativeness)**

It is measured by the usage of patterns, i.e. the frequency of the occurrence of patterns in the greedy covering, so the usage does not exceed the frequency.

It is not obvious which values are better. The high values of usage correspond to a subset of common patterns, while low values indicates that a subset contains less typical, but still interesting (w.r.t. interestingness measures) patterns.

The usage of MDL-optimal patterns is almost the same for different orders while the usage of top-n is dependent on ordering.

Used measures for ordering candidate sets.
The ordered list of candidates is used for greedy covering of data in Krimp

*area_fr_lift = frequency(B)·lift(B)*
*area_len_lift = len(B)·lift(B) = |B|·lift(B)*
*area_len_fr = len(B)·frequency(B)*

*len_fr(B)* sort by |B|,then by *frequency(B)*
*len_lift(B)* sort by |B|,then by *lift(B)*
*lift_len_fr(B)* sort by *lift(B), |B| and frequency(B)*

## References

1. Aggarwal, C.C., Han, J.: Frequent pattern mining. Springer (2014)
2. Baldi, P., Brunak, S., Chauvin, Y., Andersen, C.A., Nielsen, H.: Assessing the accuracy of prediction algorithms for classification: an overview. Bioinformatics 16(5), 412–424 (2000)
3. Buzmakov, A., Kuznetsov, S.O., Napoli, A.: Fast generation of best interval patterns for nonmonotonic constraints. In: Joint European Conference on Machine Learning and Knowledge Discovery in Databases. pp. 157–172. Springer (2015)
4. Coenen,F.:Thelucs-kdddiscretised/normalisedarmandcarmdatalibrary(2003), http://www.csc.liv.ac.uk/ frans/KDD/ Software/LUCS KDD DN
5. Ganter, B., Wille, R.: Formal concept analysis: Logical foundations (1999)
6. Ganter, B., Kuznetsov, S.O.: Formalizing hypotheses with concepts. In: International Conference on Conceptual Structures. pp. 342–356. Springer (2000)
7. Grünwald, P.D.: The minimum description length principle. MIT press (2007)
8. Kuznetsov, S.O.: Machine learning and formal concept analysis. In: Eklund, P. (ed.) Concept Lattices. Springer Berlin Heidelberg, Berlin, Heidelberg (2004)
9. Kuznetsov, S.O., Makhalova, T.: On interestingness measures of formal concepts. Information Sciences 442-443, 202 – 219 (2018)
10. Vreeken, J., Van Leeuwen, M., Siebes, A.: Krimp: mining itemsets that compress. Data Mining and Knowledge Discovery 23(1), 169–214 (2011)